Belief rule based modeling method and its application

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Outline

- Introduction to belief rule based systems
- Recursive algorithm for online updating the BRB parameters
- Sequential learning algorithm for online constructing BRB
- Hidden BRB prediction model
- Hidden behavior prediction method under testing influence





The systems are complex.

The analytic model can not be established.

Part of qualitative information and expert knowledge are available.

How to model this type of system?

Introduction to belief rule based (BRB) systems

Conventional rules vs. Belief rules

- **Conventional rules (leakage modelling):**

  IF FlowDiff is Large AND
  PressDiff is Medium
  THEN LeakSize is VeryLarge

- **Belief rules:**

  IF FlowDiff is Large AND
  PressDiff is Medium
  THEN LeakSize is {(VeryLarge, 80%), (Medium, 20%)}

Belief structure

$$\{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), ..., (D_N, \beta_{N,k})\}$$

Conventional rules vs. Belief rules

- **Single value is special case of Belief structure**

  VeryLarge <=> {(VeryLarge, 100%}

- **Conventional rules are special cases of Belief rules**
- **Probability distribution is special case of Belief structure**

$$\left(\sum_{n=1}^{N} \beta_{n,k} \le 1 \right)$$

- **Features of belief rules**
  - **Can be established according to expert knowledge**
  - **Can be trained**
  - **More accurate model**

Belief IF-THEN rules

Belief IF-THEN rule using belief structures

R_1: *IF the* ***FlowDiff*** *is* ***Large***
$\wedge$ ***PressDiff*** *is* ***Small***
THEN the ***Leaksize*** *is* **{(*Large*, 0), (*Medium*, 0.3), (*Small*, 0.7)}**

- $\wedge$ = *AND*

– **Generic belief IF-THEN rule for system modelling**

- **Subjective grade**
- **Numerical scale**
- **Event**
- **Fault**
- **……**

•**Rule R_k:**

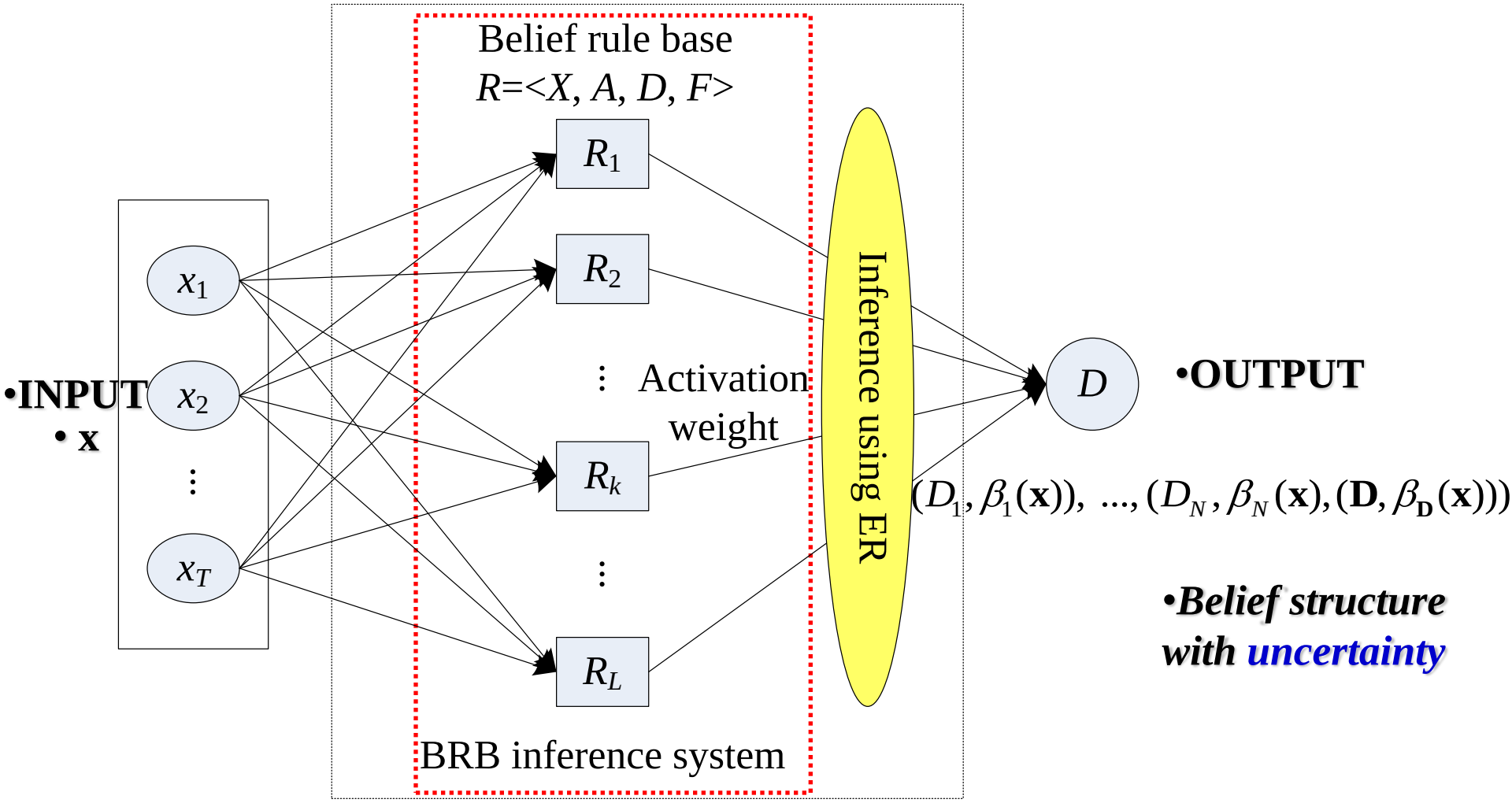
$$\text{IF } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \wedge \cdots \wedge x_{T_k} \text{ is } A_{T_k}^k,$$

$$\text{THEN } \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), ..., (D_N, \beta_{N,k})\},$$

$$\left(\sum_{n=1}^{N} \beta_{n,k} \le 1\right), \quad (n = 1, ..., N \text{ consequents}), \text{ with rule weight } \theta_k,$$

$$\text{and attribute weight } \delta_{1,k}, \delta_{2,k}, ..., \delta_{T_k,k}, \; k \in \{1, ..., L \text{ rules}\}.$$

BRB system modelling architecture



Belief rule base expression matrix

output / belief / input	D_1	D_2	.....	D_N
$A^1\ (w_1)$	$\beta_{1,1}$	$\beta_{2,1}$	$\beta_{i,1}$	$\beta_{N,1}$
⋮	⋮	⋮	⋮	⋮
$A^k\ (w_k)$	$\beta_{1,k}$	$\beta_{2,k}$	$\beta_{i,k}$	$\beta_{N,k}$
⋮	⋮	⋮	⋮	⋮
$A^L\ (w_L)$	$\beta_{1,L}$	$\beta_{2,L}$	$\beta_{i,L}$	$\beta_{N,L}$

- **The antecedent of a IF-THEN rule ⇒ *a global attribute* (A^k)**
- **An IF-THEN rule ⇒ a row in the matrix (β_{ik})**
- **An activation degree ⇒ the weight of a global attribute (w_k)**

Belief rule base for dynamic system

- Generic structure

$$R_k:\ \text{If}\ \ x_1(t)\ \text{is}\ A_1^k \wedge \cdots \wedge x_M(t)\ \text{is}\ A_M^k,\ \ \text{Then}\left\{\left(D_1,\beta_{1,k}\right),\ldots,\left(D_N,\beta_{N,k}\right)\right\}$$

With a rule weight θ_k and attribute weights $\bar{\delta}_1,\ldots,\bar{\delta}_M$

$x_1(t),\ldots,x_M(t)$: input of BRB at t time instant

$A_1^k,\ldots,A_M^k$: referential value of input

$D_1,\ldots,D_N$: referential value of output

$\beta_{1,k},\ldots,\beta_{N,k}$: belief degree

$\theta_k\ (k=1,\ldots,L)$: rule weight

$\bar{\delta}_1,\ldots,\bar{\delta}_M$: input weight

Inference using Evidential Reasoning

Step 1: The activation weight of the *k*th rule at *t* time instant

$$\omega_k(t)=\frac{\theta_k\prod_{i=1}^{M}\left(\alpha_{i,j}^{k}(t)\right)^{\bar{\delta}_i}}{\sum_{l=1}^{L}\theta_l\prod_{i=1}^{M}\left(\alpha_{i,j}^{l}(t)\right)^{\bar{\delta}_i}} \quad \text{and} \quad \bar{\delta}_i=\frac{\delta_i}{\max\limits_{i=1,\ldots,M}\{\delta_i\}}$$

Step 2:The belief rule inference using the evidential reasoning

$$\hat{\beta}_j(t)=\frac{\mu(t)\times\left[\prod_{k=1}^{L}\left(\omega_k(t)\beta_{j,k}+1-\omega_k(t)\sum_{i=1}^{N}\beta_{i,k}\right)-\prod_{k=1}^{L}\left(1-\omega_k(t)\sum_{i=1}^{N}\beta_{i,k}\right)\right]}{1-\mu(t)\times\left[\prod_{k=1}^{L}\left(1-\omega_k(t)\right)\right]}$$

$$\mu(t)=\left[\sum_{j=1}^{N}\prod_{k=1}^{L}\left(\omega_k(t)\beta_{j,k}+1-\omega_k(t)\sum_{i=1}^{N}\beta_{i,k}\right)-(N-1)\prod_{k=1}^{L}\left(1-\omega_k(t)\sum_{i=1}^{N}\beta_{i,k}\right)\right]^{-1}$$

Recursive algorithm for online updating the BRB parameters

*__Zhou Z.J.__, Hu C.H., Yang J.B., Xu D.L., Zhou D.H. Online updating belief-rule-base systems using the RIMER approach. ***IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans***. 2011, 41(6): 1225-1243.

Problem description

- **For a dynamically changing engineering system, it is difficult to obtain a complete set of historical data for developing a mathematical model for reliable system simulation and consistent decision support.**
- **In order to use belief rule base to model the complex system accurately, it is necessary to train the parameters of belief rule base.**
- **Several optimization methods have been proposed for training belief rule base. These optimization methods are implemented offline, and are not suitable for training BRB in a dynamic fashion.**
- **It is necessary to propose the recursive algorithm for online updating the BRB parameters.**

Problem description

- Generic structure

$$R_k:\ \text{If}\ \ x_1(t)\ \text{is}\ A_1^k \wedge \cdots \wedge x_M(t)\ \text{is}\ A_M^k,\ \ \text{Then}\left\{\left(D_1,\beta_{1,k}\right),\ldots,\left(D_N,\beta_{N,k}\right)\right\}$$

With a rule weight θ_k and attribute weights $\bar{\delta}_1,\ldots,\bar{\delta}_M$

- Constraints of parameters

$$0\le\theta_k\le 1,\quad k=1,\ldots,L;\qquad 0\le\bar{\delta}_m\le 1,\quad m=1,\ldots,M$$

$$0\le\beta_{j.k}\le 1;\quad \sum_{j=1}^{N}\beta_{j,k}=1,\qquad j=1,\ldots,N,k=1,\ldots,L$$

- Define

$$\mathbf{V}=\left[\theta_k,\bar{\delta}_m,\beta_{j,k}\right]^T\left(k=1,2,\ldots,L,m=1,\ldots,M,\ j=1,\ldots,N\right)$$

How to update the BRB parameter vector V recursively ?

Recursive algorithm

Step 1: **When the inputs of BRB are independent with each other, it has been proved that the outputs are independent with each other. The likelihood function can be written as**

$$f\left(\mathbf{B}(1),\ldots,\mathbf{B}(t)\middle|\mathbf{x}(1),\ldots,\mathbf{x}(t),\mathbf{Q}\right)=\prod_{\tau=1}^{t} f\left(\mathbf{B}(\tau)\middle|\mathbf{x}(\tau),\mathbf{Q}\right)$$

Step 2: **The expected likelihood function is defined as**

$$L_{t+1}(\mathbf{Q})\triangleq E\left\{\sum_{\tau=1}^{t}\log f\left(\mathbf{B}(\tau)\middle|\mathbf{x}(\tau),\mathbf{Q}\right)\middle|\mathbf{x}(1),\ldots,\mathbf{x}(t),\mathbf{Q}(t)\right\}$$

Step 3: **Recursive form:**

$$L_{t+1}(\mathbf{Q})=L_t(\mathbf{Q})+E\left\{\log f\left(\mathbf{B}(t)\middle|\mathbf{x}(t),\mathbf{Q}\right)\middle|\mathbf{x}(t),\mathbf{Q}(t)\right\}$$

Step 4: **The parameters of BRB can be online updated by**

$$\mathbf{V}(t+1)=\boldsymbol{\pi}_2\left\{\mathbf{V}(t)+\frac{\alpha}{n}\boldsymbol{\pi}_1\left\{\mathbf{V}(t)\right\}\left[\Xi\left(\mathbf{Q}(t)\right)+\gamma\mathbf{I}_{L+M+L\times N}\right]^{-1}\Gamma\left(\mathbf{Q}(t)\right)\right\}$$

Experimental verification

2008 groups of leakage data are used as testing dataset when the leak is 25%

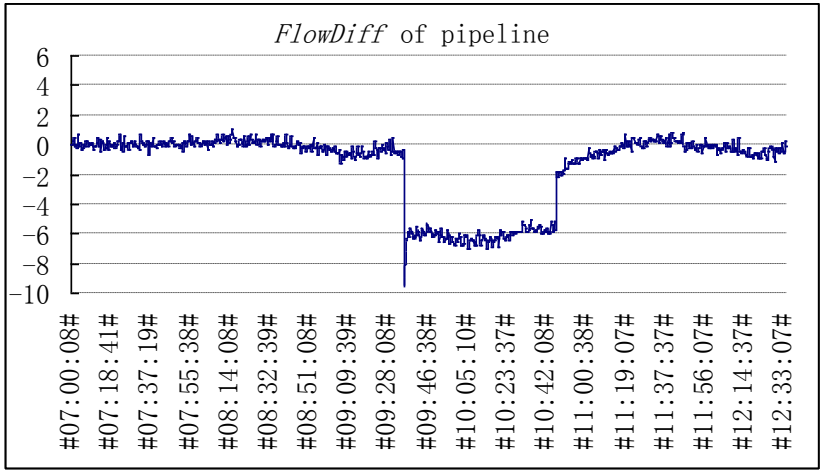


Figure: The change curve of *FlowDiff* in oil pipeline from normal to 25% leakage

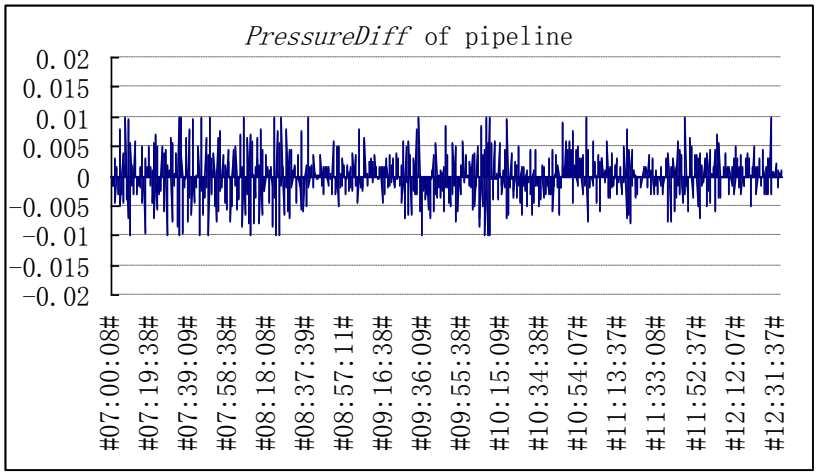


Figure: The change curve of *PressureDiff* in oil pipeline from normal to 25% leakage

$$R_k:\ \text{If}\ \ FlowDiff\ \ \text{is}\ A_1^k \wedge PressureDiff\ \ \text{is}\ A_2^k$$

$$\text{Then}\ LeakSize\ \text{is}\left\{\left(\text{Z},\beta_{1,k}\right),\left(\text{VS},\beta_{2,k}\right),\left(\text{M},\beta_{3,k}\right),\left(\text{H},\beta_{4,k}\right),\left(\text{VH},\beta_{5,k}\right)\right\}$$

$$\text{With a rule weight}\ \theta_k\ \ \text{and attribute weight}\ \delta_{1,k},\delta_{2,k},\ \left(\sum_{i=1}^{5}\beta_{i,k}\le 1\right)$$

$$A_1^k\in\{\text{NL,NM,NVS,Z,PM,PL}\}\qquad A_2^k\in\{\text{NL,NM,NS,Z,PS,PM,PL}\}$$

There are 56 belief rules in BRB for leak detection

Initial BRB system

Initial Belief Rules for Pipeline Oil Leak Detection Provided by an Expert

Rule number	FlowDiff AND PressureDiff	LeakSize distribution $\{D_1, D_2, D_3, D_4, D_5\}=\{0, 2, 4, 6, 8\}$
1	NL AND NL	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 1)\}$
2	NL AND NM	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0.3), (D_5, 0.7)\}$
3	NL AND NS	$\{(D_1, 0), (D_2, 0), (D_3, 0.2), (D_4, 0.8), (D_5, 0)\}$
4	NL AND Z	$\{(D_1, 0), (D_2, 0), (D_3, 0.8), (D_4, 0.2), (D_5, 0)\}$
5	NL AND PS	$\{(D_1, 0.65), (D_2, 0.35), (D_3, 0), (D_4, 0), (D_5, 0)\}$
6	NL AND PM	$\{(D_1, 0.85), (D_2, 0.15), (D_3, 0), (D_4, 0), (D_5, 0)\}$
7	NL AND PL	$\{(D_1, 0.95), (D_2, 0.05), (D_3, 0), (D_4, 0), (D_5, 0)\}$
8	NM AND NL	$\{(D_1, 0), (D_2, 0), (D_3, 0.1), (D_4, 0.9), (D_5, 0)\}$
9	NM AND NM	$\{(D_1, 0), (D_2, 0), (D_3, 0.7), (D_4, 0.3), (D_5, 0)\}$
10	NM AND NS	$\{(D_1, 0), (D_2, 0.7), (D_3, 0.3), (D_4, 0), (D_5, 0)\}$
11	NM AND Z	$\{(D_1, 0), (D_2, 0.9), (D_3, 0.1), (D_4, 0), (D_5, 0)\}$
12	NM AND PS	$\{(D_1, 0.8), (D_2, 0.2), (D_3, 0), (D_4, 0), (D_5, 0)\}$
13	NM AND PM	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 0)\}$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
54	PL AND PS	{(D1, 1), (D2, 0), (D3, 0), (D4, 0), (D5, 0)}
55	PL AND PM	{(D1, 1), (D2, 0), (D3, 0), (D4, 0), (D5, 0)}
56	PL AND PL	{(D1, 1), (D2, 0), (D3, 0), (D4, 0), (D5, 0)}

Updated BRB system

800 data sets are selected from 2008 leak data and used as training data.

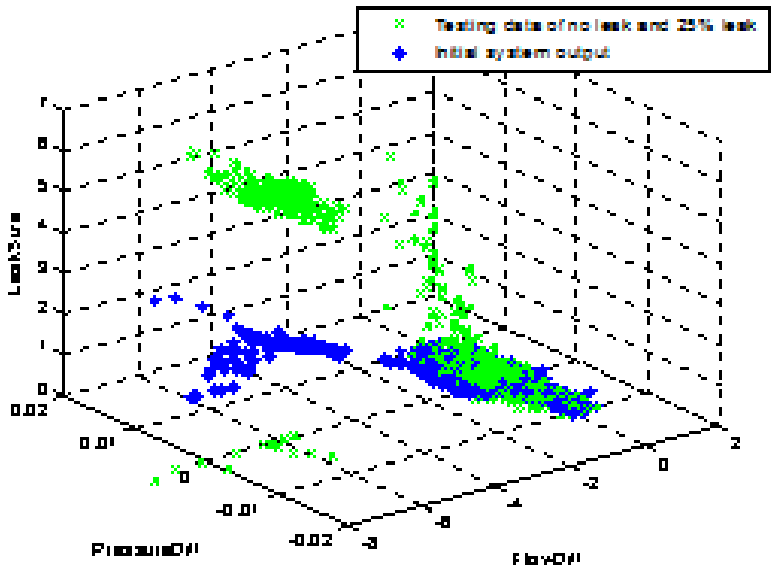


Figure. Testing data of no leak and 25% leak and outputs generated by the initial BRB

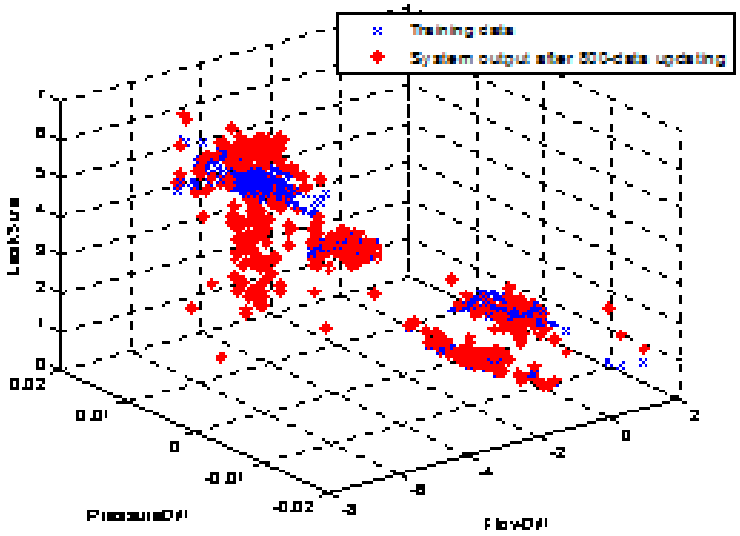


Figure. Training data and outputs generated by the updated BRB

The updated BRB can closely replicate the relationship among *FlowDiff*, *PressureDiff* and *LeakSize* in the training data.

Updated BRB system

Updated Belief Rules and Rule Weights

Rule number	Updated rule weight	FlowDiff AND PressureDiff	LeakSize distribution $\{D_1, D_2, D_3, D_4, D_5\}=\{0, 2, 4, 6, 8\}$
1	1	NL AND NL	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 1)\}$
2	1	NL AND NM	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0.3), (D_5, 0.7)\}$
3	1	NL AND NS	$\{(D_1, 0.001), (D_2, 0.001), (D_3, 0.0473), (D_4, 0.4727), (D_5, 0.4780)\}$
4	0.7669	NL AND Z	$\{(D_1, 0.0061), (D_2, 0.0075), (D_3, 0.0106), (D_4, 0.5417), (D_5, 0.4341)\}$
5	0.8852	NL AND PS	$\{(D_1, 0.0079), (D_2, 0.0092), (D_3, 0.001), (D_4, 0.4399), (D_5, 0.542)\}$
6	1	NL AND PM	$\{(D_1, 0.85), (D_2, 0.15), (D_3, 0), (D_4, 0), (D_5, 0)\}$
7	1	NL AND PL	$\{(D_1, 0.95), (D_2, 0.05), (D_3, 0), (D_4, 0), (D_5, 0)\}$
8	0.9616	NM AND NL	$\{(D_1, 0.1619), (D_2, 0.0771), (D_3, 0.1005), (D_4, 0.6595), (D_5, 0.0009)\}$
9	1	NM AND NM	$\{(D_1, 0.1641), (D_2, 0.0865), (D_3, 0.5916), (D_4, 0.1566), (D_5, 0.0011)\}$
10	0.8619	NM AND NS	$\{(D_1, 0.0707), (D_2, 0.0392), (D_3, 0.0303), (D_4, 0.3982), (D_5, 0.4615)\}$
11	0.562	NM AND Z	$\{(D_1, 0.0678), (D_2, 0.0786), (D_3, 0.0131), (D_4, 0.4946), (D_5, 0.346)\}$
12	0.9196	NM AND PS	$\{(D_1, 0.0142), (D_2, 0.0048), (D_3, 0.0079), (D_4, 0.486), (D_5, 0.4871)\}$
13	0.955	NM AND PM	$\{(D_1, 0.4595), (D_2, 0.0128), (D_3, 0.0014), (D_4, 0.179), (D_5, 0.3473)\}$
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
54	1	PL AND PS	{(D1, 1), (D2, 0), (D3, 0), (D4, 0), (D5, 0)}
55	1	PL AND PM	{(D1, 1), (D2, 0), (D3, 0), (D4, 0), (D5, 0)}
56	1	PL AND PL	{(D1, 1), (D2, 0), (D3, 0), (D4, 0), (D5, 0)}

Test of updated BRB system

For testing the updated BRB, all the 2008 data are used to test the updated BRB system.

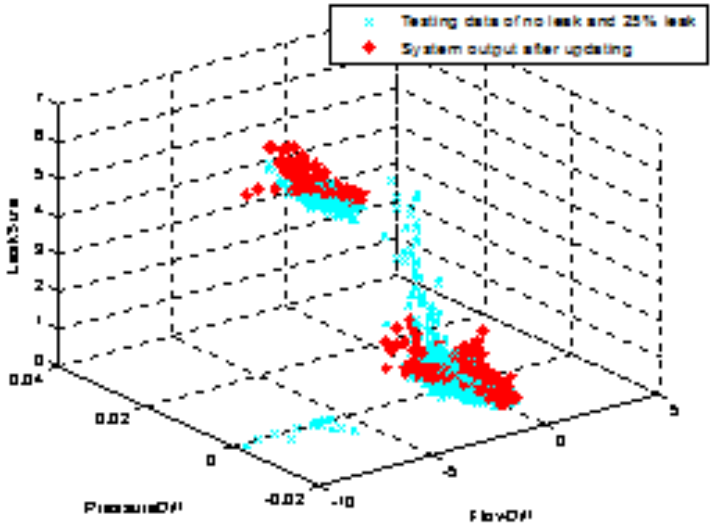


Figure: The testing data of no leakage and 25% leakage and the leakage estimations generated by updated BRB

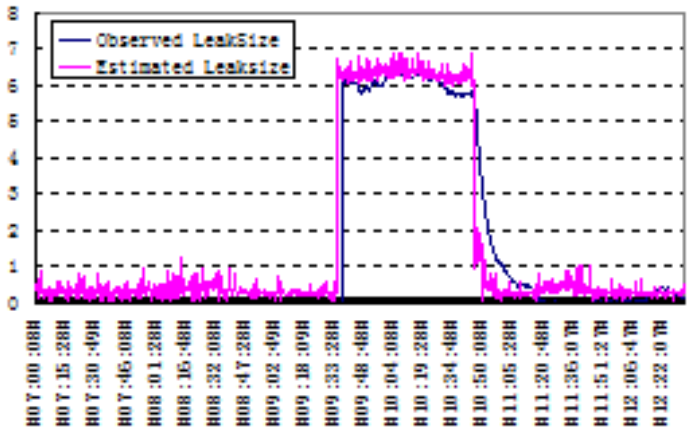


Figure: The testing data of no leakage and 25% leakage and the leakage estimations generated by updated BRB

- **It shows that rule base can detect the leak which happened at around 9:34 a.m. and ended at around 10:50 a.m.**
- **It can also be concluded that the proposed approach can be widely applied in engineering.**

Sequential learning algorithm for online constructing BRB

•**Zhou Z.J.**, Hu C.H., Yang J.B., Xu D.L., Chen M.Y., Zhou D.H. A sequential learning algorithm for online constructing belief-rule-based systems. ***Expert Systems with Applications***. 2010, 37(2): 1790-1799.

Problem description

In engineering practice, when the prior knowledge is finite, the initial belief rule base given by experts is incomplete for some complex dynamic systems. On the contrary, too much prior knowledge may lead to too much rules in the initial rule base.

1) How to effectively use the observation information to online test the existing rules of BRB, and determine whether it should be deleted according to the test results.

2) How to effectively use the observation information to online add some new rules for the BRB system.

Problem description

- Generic structure

$$R_k:\ \text{If}\ \ x_1(t)\ \text{is}\ A_1^k \wedge \cdots \wedge x_M(t)\ \text{is}\ A_M^k,\ \ \text{Then}\left\{\left(D_1,\beta_{1,k}\right),\ldots,\left(D_N,\beta_{N,k}\right)\right\}$$

With a rule weight θ_k and attribute weights $\bar{\delta}_1,\ldots,\bar{\delta}_M$

How to construct a compact BRB?

It is necessary to propose a novel way to estimate the structure and the parameters of a BRB system at same time, which is very important for the BRB to achieve the overall optimal performanc

In other words, the proposed algorithm should identify the structure and parameter of BRB.

Statistical utility of belief rule

Statistical utility

$$\lim_{n\to+\infty} U(k) = \sum_{j=1}^{N} u(D_j)\beta_{j,k} \frac{\lim_{n\to+\infty} \sum_{t=1}^{n} c_k(\mathbf{x}(t)) \Big/ n}{\sum_{l=1}^{L} \lim_{n\to+\infty} \sum_{t=1}^{n} c_l(\mathbf{x}(t)) \Big/ n}$$

The statistical utility of a belief rule gives a measure of the information content in a belief rule and the contribution made by that belief rule to the BRB output over all the input data received so far.

Online constructing BRB

- **The online construction of BRB includes real-time identification of structures and online parameter estimation.**
- **Real-time identification of structures includes adding a new belief rule and pruning of a belief rule from the current belief rule base according to the new information.**
- **When the structure of belief rule base is determined, it is necessary to update the parameters online.**
- **This is very helpful to produce a more compact BRB and reduce the system complexity. Moreover, the overall performance of BRB to model the complex system can be ensured.**

Adding a belief rule

If the statistical utility of a belief rule is large, it is logical to say that this rule is significant and should be added to a BRB.

$$\begin{cases} \left\| \hat{\mathbf{x}}(n) - \overline{\boldsymbol{\gamma}}(n) \right\| > \varepsilon \\ \dfrac{\hat{\mathbf{y}}(n) E_{L+1}\left(\boldsymbol{\gamma}_{L+1}\right)}{\sum_{l=1}^{L+1} E_l\left(\boldsymbol{\gamma}_l\right)} > e_g \end{cases}$$

1) The distance threshold ε is set to around 10% of the upper bound of input variables.

2) The adding threshold e_g is chosen according to the desired accuracy.

The first criterion requires that a new belief rule may be added if the input data is sufficiently far from the existing belief rule (in the Euclidean distance sense).

The second one requires that the statistical utility of a newly added belief rule is greater than a given approximation accuracy.

Pruning of a belief rule

If an activated belief rule is insignificant in the BRB, it should be pruned.

$$\frac{E_k(\gamma_k)\sum_{j=1}^{N} u(D_j)\beta_{j,k}}{\sum_{l=1}^{L} E_l(\gamma_l)} < e_p$$

In general, the pruning threshold e_p is set to around 10% of e_g.

Obviously, if ε and e_g are small, the system performance will be better, but the resulting BRB's structure is more complex, which is a disadvantage when there is high real-time requirement.

Therefore, we should choose these parameters carefully according to the expected system performance.

Updating the BRB parameters

Recursive learning algorithm for online updating BRB parameters

$$\mathbf{Q}(n+1)=\prod\nolimits_{H_2}\left\{\mathbf{Q}(n)+\frac{1}{n}\left[\Xi_2\left(\mathbf{Q}(n)\right)\right]^{-1}\Gamma_2\left(\mathbf{Q}(n)\right)\right\}$$

$$\Gamma_2\left(\mathbf{Q}(n)\right)\triangleq\nabla_{\mathbf{Q}}\log f\left(\hat{\mathbf{B}}(n)\middle|\hat{\mathbf{x}}(n),\mathbf{Q}(n)\right)$$

$$\Xi_2\left(\mathbf{Q}(n)\right)\triangleq E\left\{-\nabla_{\mathbf{Q}}\nabla_{\mathbf{Q}}{}^{T}\log f\left(\hat{\mathbf{B}}(n)\middle|\hat{\mathbf{x}}(n),\mathbf{Q}\right)\middle|\hat{\mathbf{x}}(n),\mathbf{Q}(n)\right\}$$

1）A belief rule is added or pruned automatically and also the parameters of the BRB system are updated using the training data sequentially (one by one).

2）When new information becomes available, the algorithm immediately adjusts the structure and the parameters of the BRB without having to wait for all information to be provided.

3）The statistical utility of a belief rule directly links the required learning accuracy to the utility of a belief rule in the learning algorithm so as to realize a compact BRB system.

Experimental verification

Initial BRB for leak detection

Table Initial BRB given by experts

规则编号	*FlowDiff* AND *PressureDiff*	*LeakSize* 分布式评价结果 $\{D_1, D_2, D_3, D_4, D_5\}=\{0, 2, 4, 6, 8\}$
1	NL AND NL	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 1)\}$
2	PL AND PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$

Online update BRB

（900 groups of leakage data are used as training dataset）

Table Updated BRB

规则编号	更新的规则权重	*FlowDiff* AND *PressureDiff*	*LeakSize* 分布式评价结果 $\{D_1, D_2, D_3, D_4, D_5\}=\{0, 2, 4, 6, 8\}$
1	0.0009	NL AND NL	$\{(D_1, 0.2785), (D_2, 0.0372), (D_3, 0), (D_4, 0.0628), (D_5, 6214)\}$
2	1.0000	NM AND Z	$\{(D_1, 0.0566), (D_2, 0.0288), (D_3, 0.0339), (D_4, 0.7423), (D_5, 0.1384)\}$
3	1.0000	NS AND Z	$\{(D_1, 0.6768), (D_2, 0.3198), (D_3, 0.0014), (D_4, 0.0010), (D_5, 0.0010)\}$
4	0.6714	NS AND PS	$\{(D_1, 0.6750), (D_2, 0.2649), (D_3, 0.0003), (D_4, 0.0588), (D_5, 0.0010)\}$
5	1.0000	PL AND PL	$\{(D_1, 6920), (D_2, 0.2988), (D_3, 0.0082), (D_4, 0.0001), (D_5, 0.0010)\}$

Online update BRB

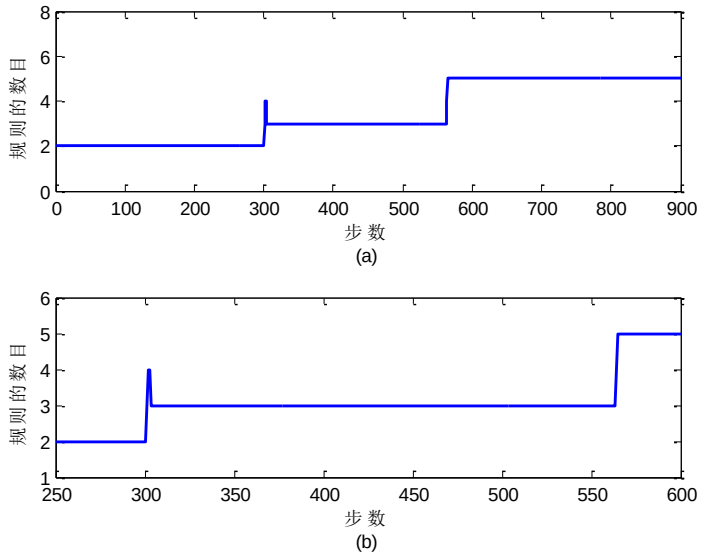


Figure: The change curve of the number of belief rules in the BRB updating

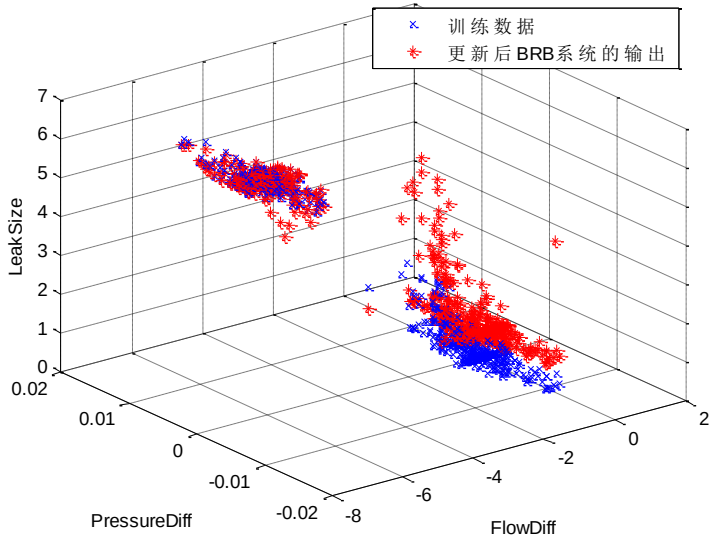


Figure: Training data and output estimation values generated by updated BRB system

1）From the first Figure, we can see that the number of the belief rules changes with time, and reaches a fixed value at last.

2）From the second Figure, it can seen that the updated BRB can closely replicate the relationship among *FlowDiff*, *PressureDiff* and *LeakSize* in the training data.

Test the updated BRB

2008 groups of leakage data are used as testing dataset

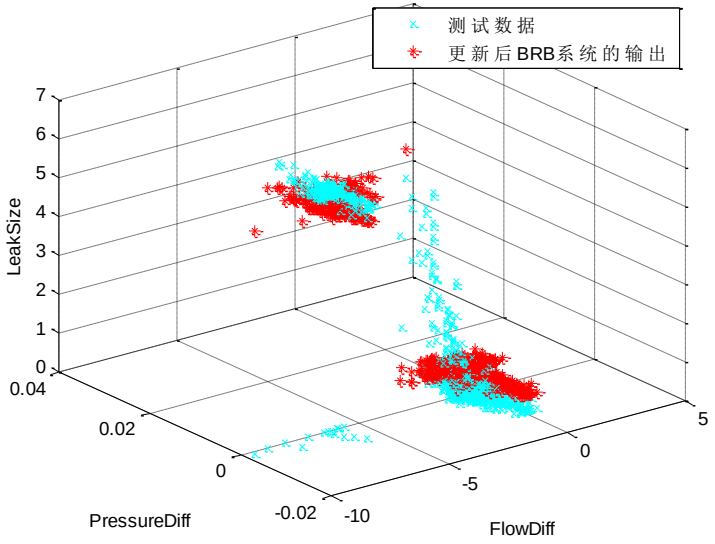


Figure. Testing data and output estimation values generated by updated BRB system

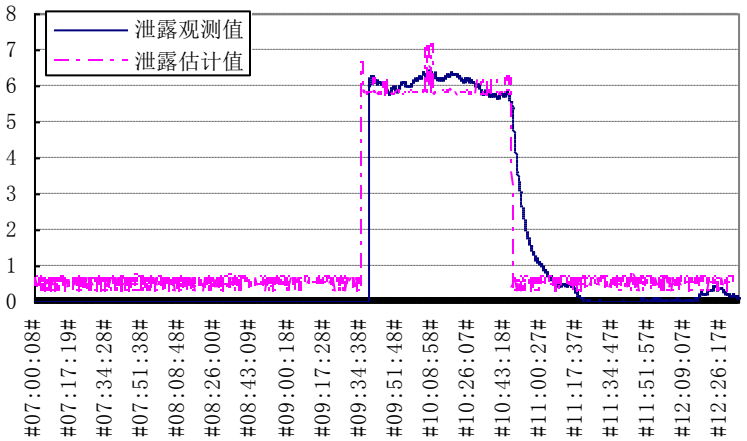


Figure. Testing data in time-scale and output estimation values generated by updated BRB system

It can be concluded that the newly constructed BRB can be used to detect oil pipeline leak.

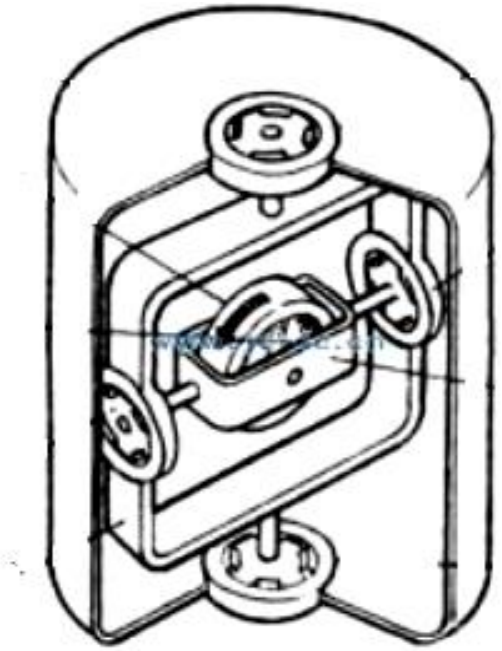
Hidden BRB prediction model

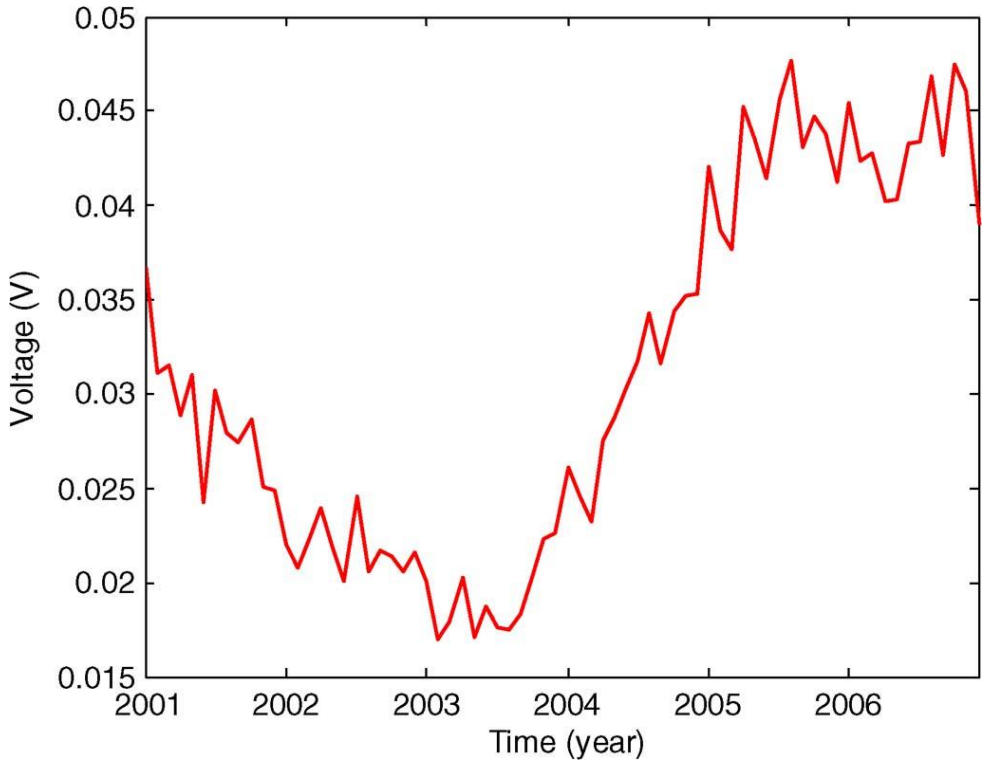
*__Zhou Z.J.,__ Hu C.H., Zhang B.C., Xu D.L., Chen Y.W. Hidden behavior prediction of complex systems based on hybrid information. ***IEEE Transactions on Cybernetics***. 2013, 43(2): 402-411.

Problem description

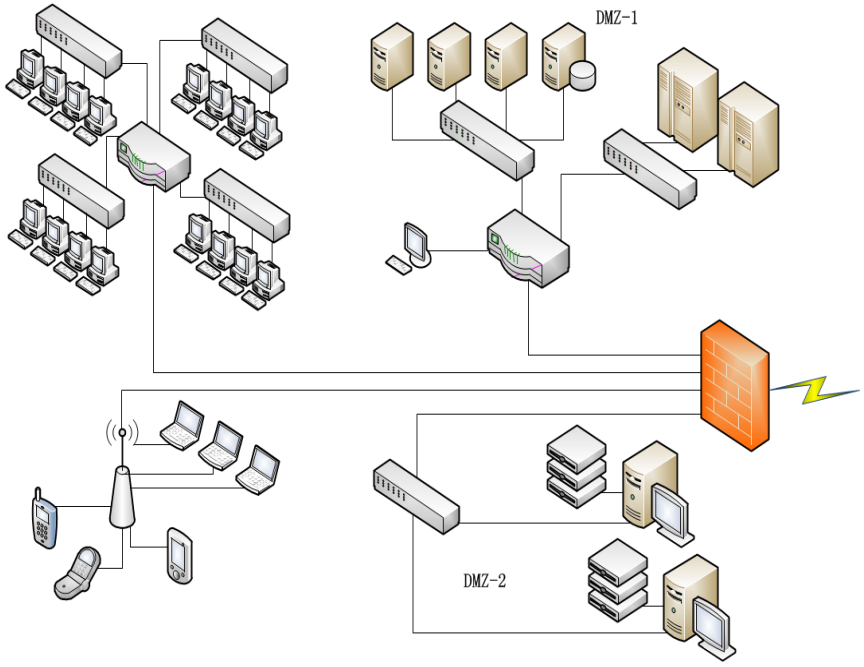
It is very important to predict system behaviors. For example, in some systems such as nuclear power stations and rocket control loops that have high requirements for safety, it is necessary to predict the system behaviors that may lead to major faults. In engineering practice, the system behaviors can be divided into two types: the first type is the behavior that can be observed directly, and the second type is that is hidden and can be observed indirectly.

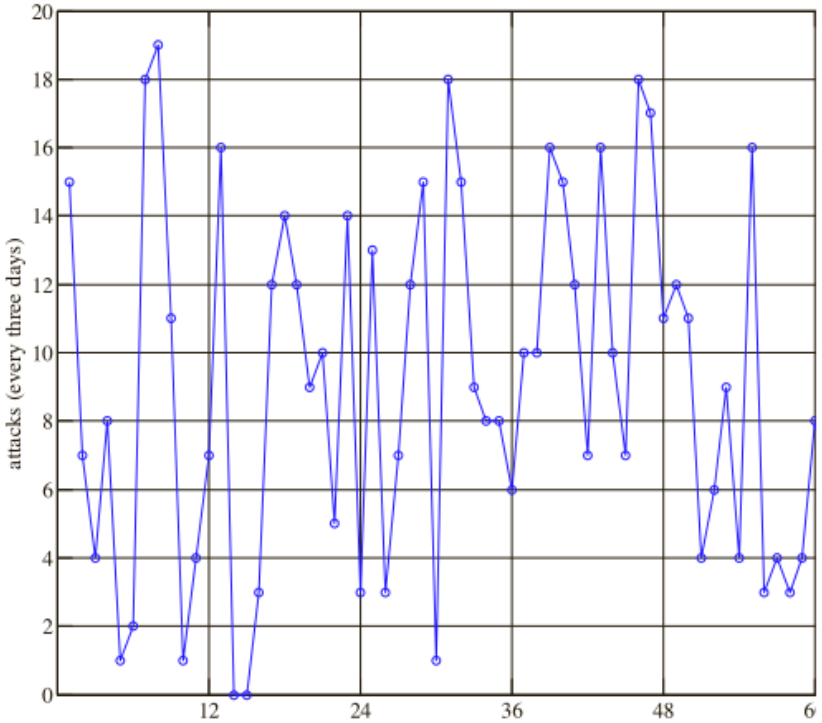
Gyro is an important device in the inertial platform system, and drift is the main factor of affecting the performance of gyro. However, when the inertial platform works, the gyro drift is not directly observable and can only be assessed via other measured condition data such as voltage or electricity.





Network security situation which can reflect the network security status can be also considered as a hidden behavior of complex network system. The network security situation can be measured by some other security factors such as the attack type and the attack strength.





Problem description

Although belief rule base (BRB) has been employed to predict the observable behavior using the hybrid information with uncertainties, it is still not applicable to predict the hidden behavior directly. As such, a new BRB model is proposed to predict the hidden behavior.

How to predict the hidden behavior of complex system through the exsting observable behavior ?

Establishing Hidden BRB

In order to capture the relationship between the behaviors at current time instant t and future time instant (t + 1), a BRB can be constructed as follows:

$$R_k : \text{If } x(t) \text{ is } D_k,$$
$$\text{Then } x(t+1) \text{ is } \{(D_1,\beta_{1,k}),\ldots,(D_N,\beta_{N,k}),(D,\beta_{D,k})\}$$
$$\text{With rule weight } \theta_k \text{ and attribute weight } \delta_k$$

It is assumed that the hidden behavior x(t) can be reflected indirectly by y(t), and the relationship between x(t) and y(t) is described as follows

$$\mathbf{y}(t) = h\left(\boldsymbol{\psi}_2, x(t), \boldsymbol{\nu}(t)\right)$$

The key is to estimate the parameters in belief rule base and observable equation.

Updating the BRB parameters

- **Step 1: Construction of Likelihood Function**

When the dimension of observation y(t) is one, the vector y(t) is simplified as a scalar quantity y(t). Let Y (t) = [y(1),...,y(t)] and the following likelihood function can be constructed:

$$L(\mathbf{Q}) = \prod_{t=2}^{T} p\left(y(t)|Y(t-1)\right)$$

● Step 2: Construction of the objective function

When the likelihood function is obtained, the parameters of forecasting model can be estimated by solving the following nonlinear opti

$$\max_{\mathbf{Q}} L(\mathbf{Q})$$

$$\text{s.t. } 0 \le \theta_k \le 1,\ k = 1, \ldots, N,$$

$$0 \le \beta_{j,k} \le 1,\ j = 1, \ldots, N,\ k = 1, \ldots, N,$$

$$0 \le \beta_{D,k} \le 1,\ k = 1, \ldots, N,$$

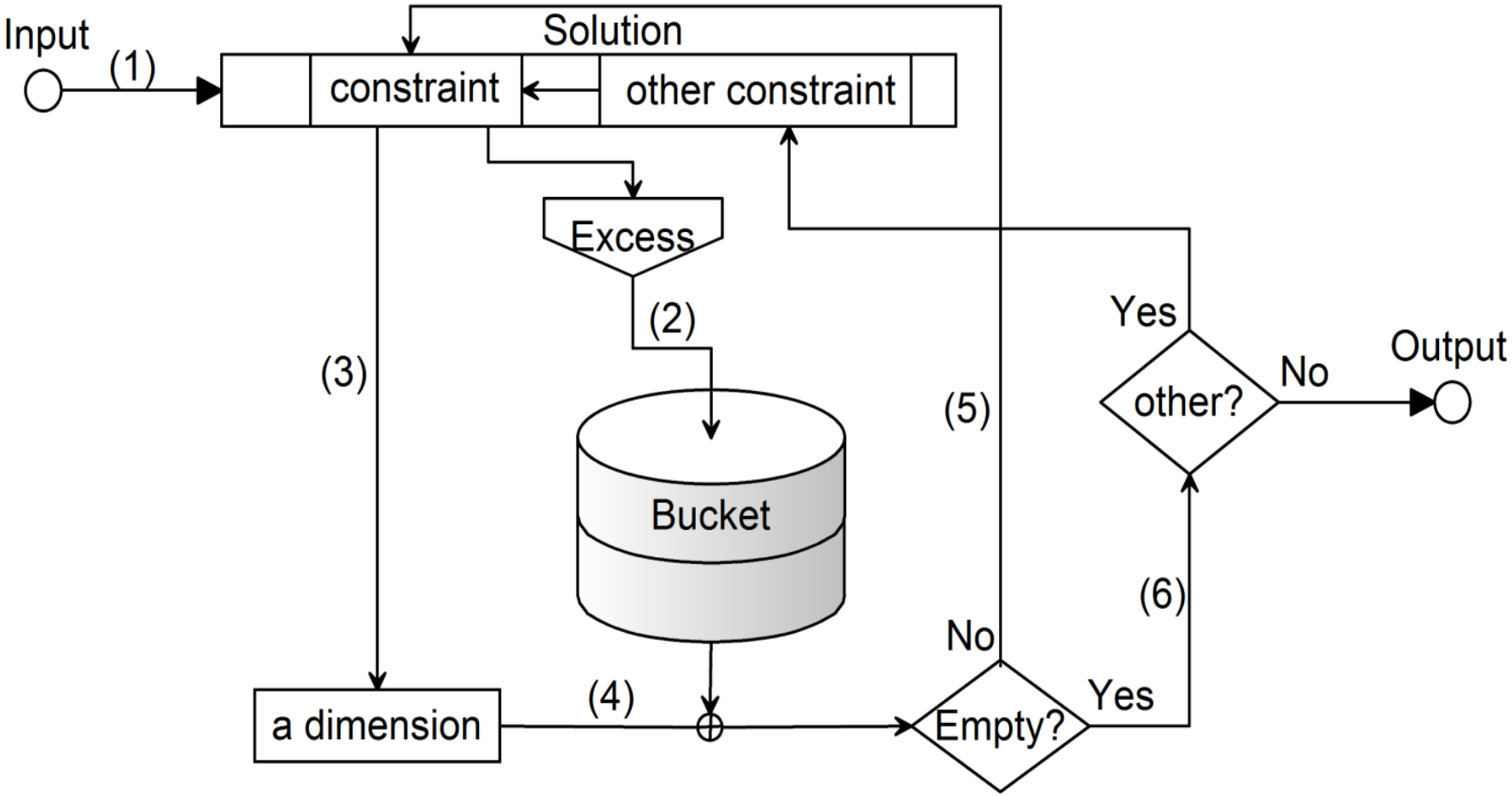
$$\beta_{D,k} + \sum_{j=1}^{N} \beta_{j,k} = 1,$$

$$c(\boldsymbol{\psi_2}) \le 0,$$

$$ceq(\boldsymbol{\psi_2}) = 0$$

- **Step 3: The constraint CMA-ES algorithm is used to solve the optimization model**

Three operations are included: sampling operation, selection operation, and adapting operation.



Experimental verification

To demonstrate that the HBRB can be applied in practice, a case study is conducted in which 72 testing data of voltage and 72 testing data of the zero-item drift parameter from 2001 to 2006 are selected.

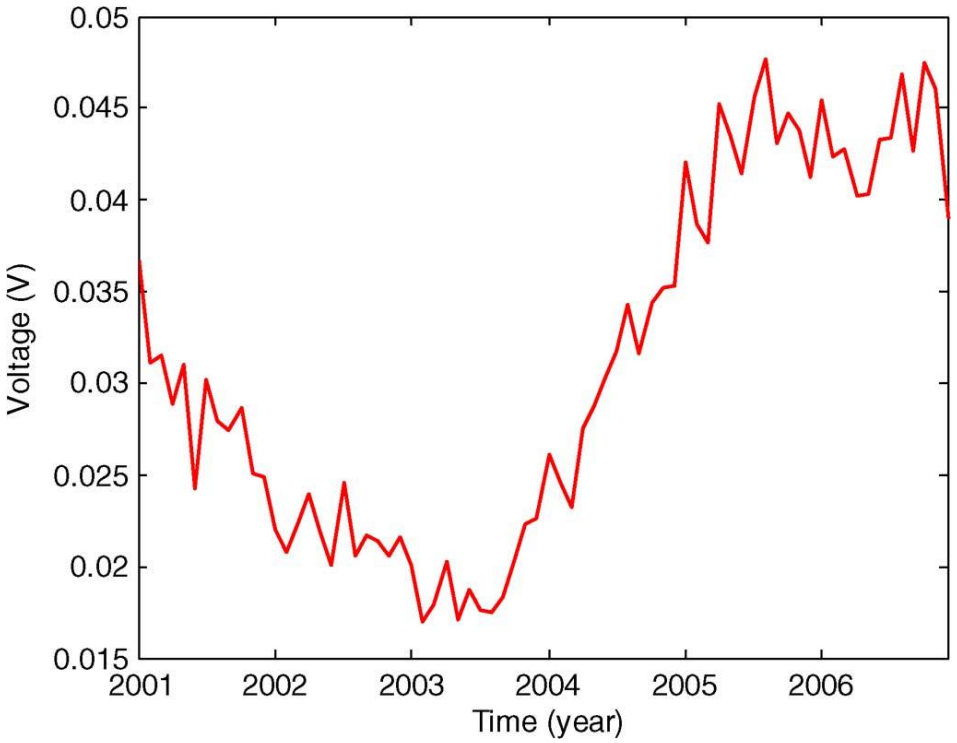


Figure. Testing data of voltage

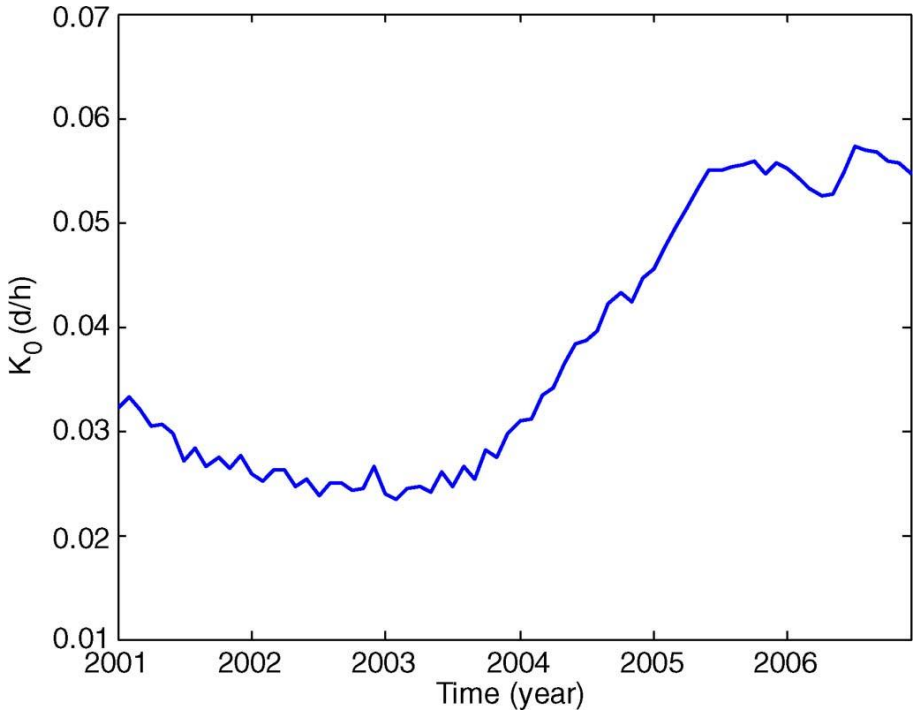


Figure. Testing data of zero-item drift parameter

In order to construct HBRB for forecasting the zero-item drift parameter K0, three referential points is assigned to K0 and they are small (S), medium (M), and large (L). The initial parameters given by experts are shown in Tables 1 and 2.

Table 1. REFERENTIAL POINTS OF K0

Linguistic terms	S	M	L
Numerical values	0.02	0.04	0.06

Table 2. INITIAL BELIEF DEGREES

Rule number	$K_0(t)$	$K_0(t+1)$ distribution $\{D_1, D_2, D_3\}=\{S, M, L, D\}$
1	S	$\{(D_1, 0.9), (D_2, 0.05), (D_3, 0.03), (D, 0.02)\}$
2	M	$\{(D_1, 0.8), (D_2, 0.14), (D_3, 0.05), (D, 0.01)\}$
3	L	$\{(D_1, 0.6), (D_2, 0.3), (D_3, 0.08), (D, 0.02)\}$

When the HBRB model is established with the above initial parameters, the constraint CMA-ES is used to optimize the parameters. The optimal parameters are shown in Table 3.

Table 3. TRAINED RULE WEIGHTS AND BELIEFD EGREES

Rule number	Trained rule weight	$K_0(t)$	$K_0(t+1)$ distribution $\{D_1, D_2, D_3, D\}=\{S, M, L, D\}$
1	0.4083	S	$\{(D_1, 0.8596), (D_2, 0.0162), (D_3, 0.0260), (D, 0.0982)\}$
2	0.8348	M	$\{(D_1, 0.0670), (D_2, 0.8834), (D_3, 0.0340), (D, 0.0156)\}$
3	0.9653	L	$\{(D_1, 0.0210), (D_2, 0.1247), (D_3, 0.7928), (D, 0.0615)\}$

The following figure gives the first 67 values of zero-item drift parameter and the estimates generated by the updated forecasting model. It shows that the updated model can predict the first 67 values of zero-item drift parameter accurately.

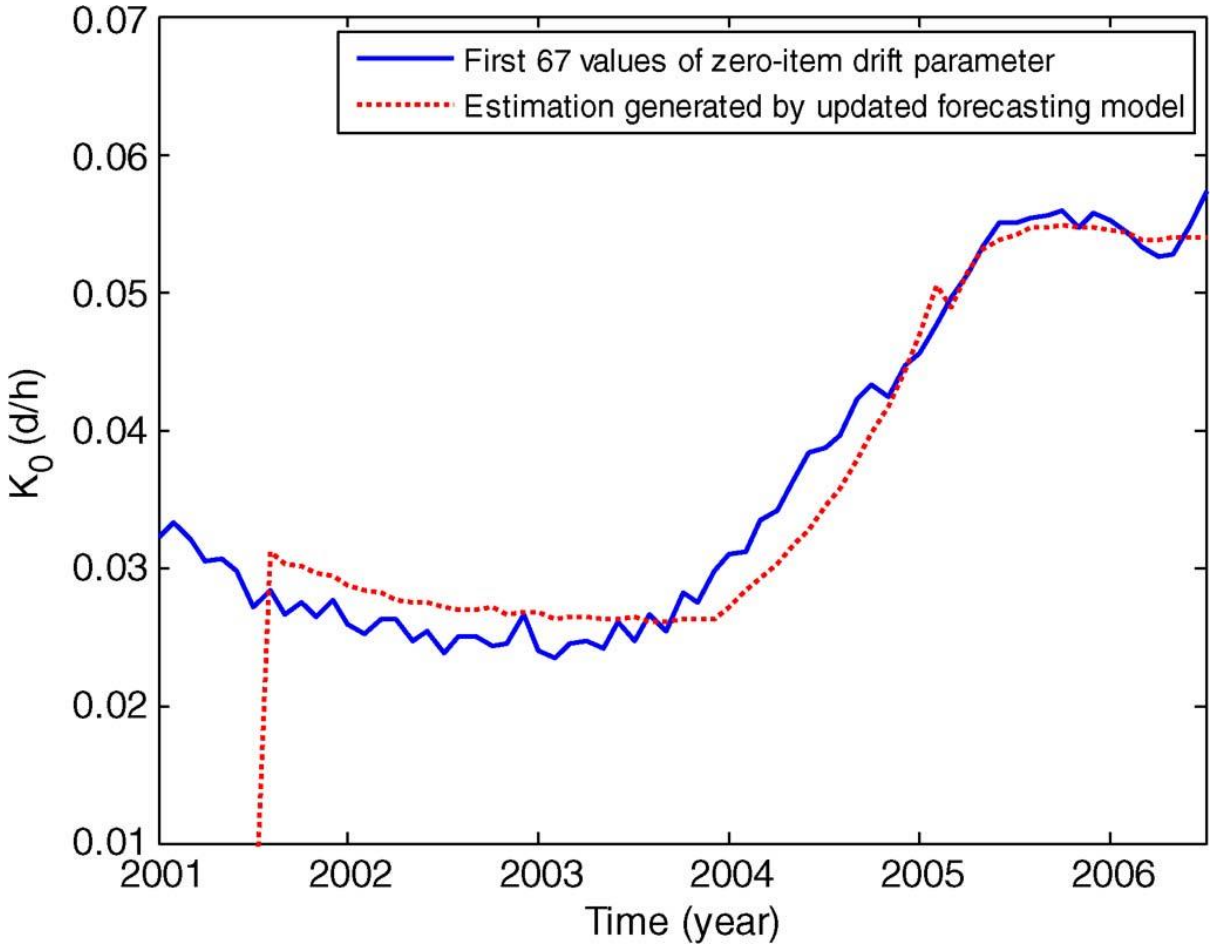


Figure. First 67 values of zero-item drift parameter and the estimates generated by updated forecasting model

Hidden behavior prediction method under testing influence

*__Zhou Z.J.__, Hu C.H., Hu G.Y., Han X.X., Zhang B.C., Chen Y.W. Hidden behavior prediction of complex systems under testing influence based on semi-quantitative information and belief rule base. ***IEEE Transactions on Fuzzy Systems***, vol. 23, no. 6, pp. 2371-2386, 2015.

Problem description

1）The systems can not be tested periodically and the observable information is incomplete.

2）The change process of hidden behavior may be affected by the test.

3）Only part of quantitative information and qualitative knowledge (i.e., semi-quantitative information) may be obtained.

These characteristics all related to the test are named as testing influence for simplicity.

Although a model and a corresponding optimal algorithm for training the model parameters have been proposed to predict the hidden behavior on the basis of semi-quantitative information and belief rule base (BRB), the testing influence has not been considered. In order to solve the above problems, a new BRB based model, which can use the semi-quantitative information, is proposed under testing influence.

Prediction model under testing influence

Prediction model

$$R_k: \text{If } x(t-1) \text{ is } A_1^k \wedge I(t) \text{ is } A_2^k,$$

$$\text{Then } x(t) \text{ is } \left\{\left(D_1, \beta_{1,k}\right), \ldots, \left(D_N, \beta_{N,k}\right)\right\}$$

With rule weight θ_k and attribute weights δ_1, δ_2

Testing influence

$$\begin{cases} I(t) = 0, & \text{test is not carried out} \\ I(t) = 1, & \text{test is carried out} \end{cases}$$

Observation equation

$$\mathbf{y}(t) = h\left(\boldsymbol{\psi}_2, x(t)\right) + \mathbf{v}(t)$$

It can be concluded that a part of testing data (i.e., observation) may be missed and the observable information is incomplete.

Updating the BRB parameters

Step 1: Construction of objective function

a) Conditional probability density

$$f\left(Y(t)\middle|\mathbf{Q}\right)=f\left(Y_{\text{obs}}(t),Y_{\text{mis}}(t)\middle|\mathbf{Q}\right)$$
$$=f\left(Y_{\text{obs}}(t)\middle|\mathbf{Q}\right)f\left(Y_{\text{mis}}(t)\middle|Y_{\text{obs}}(t),\mathbf{Q}\right)$$

b) Likelihood function

$$l\left(\mathbf{Q}\middle|Y_{\text{obs}}(t)\right)=l\left(\mathbf{Q}\middle|Y(t)\right)-\ln f\left(Y_{\text{mis}}(t)\middle|Y_{\text{obs}}(t),\mathbf{Q}\right)$$
$$l\left(\mathbf{Q}\middle|Y_{\text{obs}}(t)\right)=W\left(\mathbf{Q}\middle|\mathbf{Q}(t)\right)-H\left(\mathbf{Q}\middle|\mathbf{Q}(t)\right)$$

c) Expectation

$$l\left(\mathbf{Q}\middle|Y_{\text{obs}}(t)\right)=W\left(\mathbf{Q}\middle|\mathbf{Q}(t)\right)$$

d) The objective function

$$W\left(\mathbf{Q}\middle|\mathbf{Q}^m\right)=\int\left(-T\ln 2\pi-\frac{T}{2}\ln\Sigma-\frac{1}{2}\sum_{t=1}^{T}\left(\mathbf{y}(t)-h\left(\boldsymbol{\psi}_2,x(t)\right)\right)'\Sigma^{-1}\left(\mathbf{y}(t)-h\left(\boldsymbol{\psi}_2,x(t)\right)\right)\right)f\left(Y_{\text{mis}}\middle|Y_{\text{obs}},\mathbf{Q}^m\right)dY_{\text{mis}}$$
$$=-T\ln 2\pi-\frac{T}{2}\ln\Sigma-\frac{1}{2}\sum_{t=1}^{T}E\left(\mathbf{y}(t)'\Sigma^{-1}\mathbf{y}(t)\middle|Y_{\text{obs}},\mathbf{Q}^m\right)+\sum_{t=1}^{T}E\left(\mathbf{y}(t)'\Sigma^{-1}h\left(\boldsymbol{\psi}_2,x(t)\right)\middle|Y_{\text{obs}},\mathbf{Q}^m\right)-\frac{1}{2}\sum_{t=1}^{T}h\left(\boldsymbol{\psi}_2,x(t)\right)'\Sigma^{-1}h\left(\boldsymbol{\psi}_2,x(t)\right)$$
$$=-T\ln 2\pi-\frac{T}{2}\ln\Sigma-\frac{1}{2}\sum_{t=1}^{T}E\left(\mathbf{y}(t)'\Sigma^{-1}\mathbf{y}(t)\middle|Y_{\text{obs}},\mathbf{Q}^m\right)+\sum_{t=1}^{T}h\left(\boldsymbol{\psi}_2,x(t)\right)'\Sigma^{-1}E\left(\mathbf{y}(t)\middle|Y_{\text{obs}},\mathbf{Q}^m\right)-\frac{1}{2}\sum_{t=1}^{T}h\left(\boldsymbol{\psi}_2,x(t)\right)'\Sigma^{-1}h\left(\boldsymbol{\psi}_2,x(t)\right)$$

Step 2: parameter optimization

In order to train the parameters of the BRB model where the testing influence is considered, an objective function is constructed on the basis of the idea of expectation-maximization (EM) which is appropriate to estimate the parameters when the observations are missed.

$$\max\left\{W\left(\mathbf{Q}\middle|\mathbf{Q}^{m}\right)\right\}$$

$$\text{s.t.}\quad 0\le\theta_k\le 1,\quad k=1,\ldots,L,$$

$$0\le\beta_{j,k}\le 1,\quad j=1,\ldots,N,k=1,\ldots,L,$$

$$\sum_{j=1}^{N}\beta_{j,k}=1,$$

The above function is continuous nonlinear optimization problem with equality and inequality constraints. An improved CMA-ES algorithm is developed further in order to obtain the optimal values of the parameters.

Experimental verification

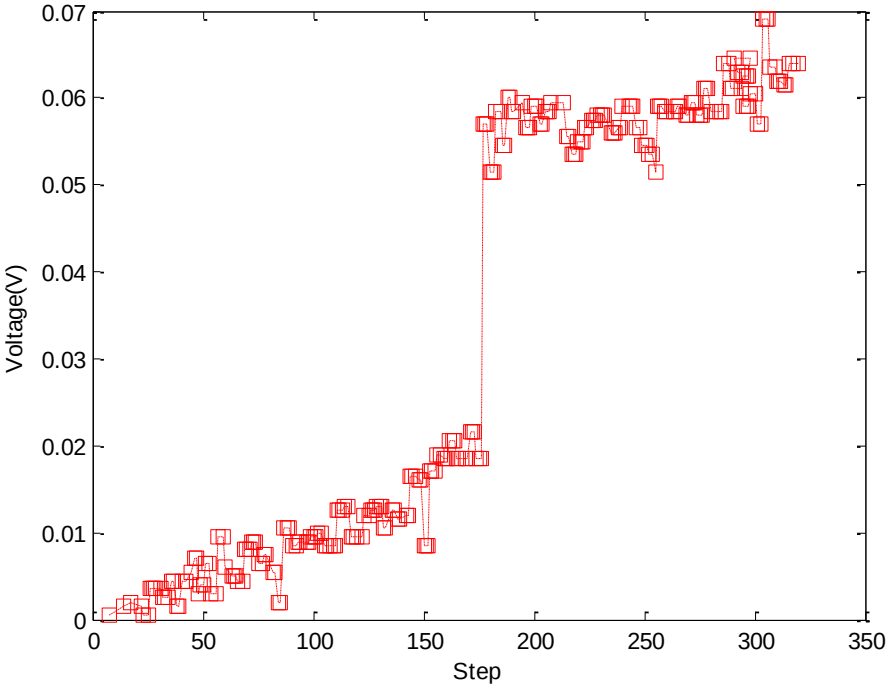


Figure. testing data of voltage under testing influence

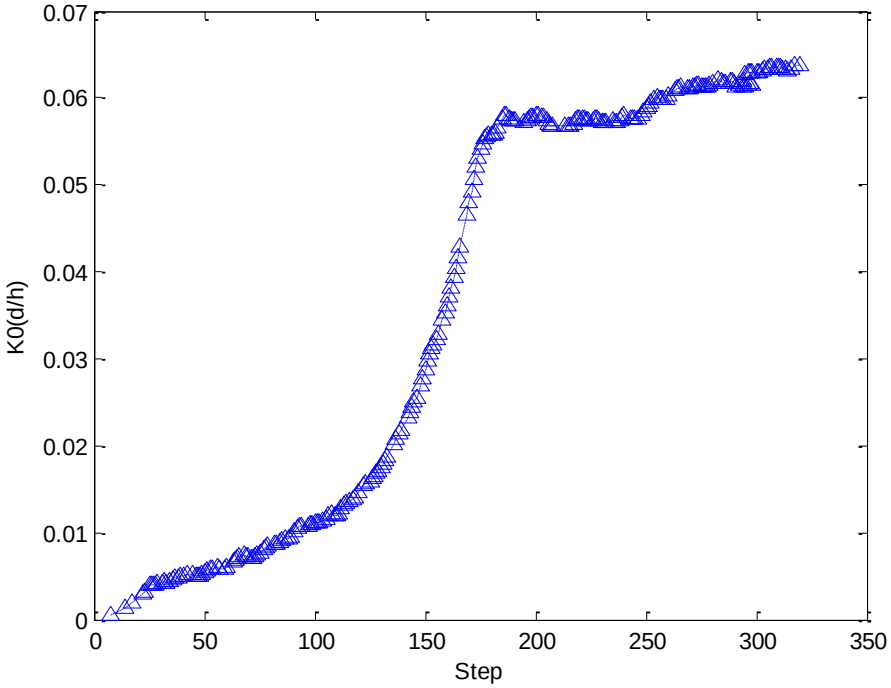


Figure. testing data of zero-item drift parameter under testing influence

- **The above figures give the testing data of voltage and the corresponding testing data of the zero-item drift parameter.**

The initial belief degrees of BRB as shown in Table 1.

The trained parameters are shown in Table 2.

Table 1. Initial BRB under testing influence

Rule number	$K_0(t\text{-}1)$ and $I(t)$	$K_0(t)$ distribution $\{D_1, D_2, D_3\}=\{S, M, L\}$
1	S and 0	$\{(D_1, 0.99), (D_2, 0.01), (D_3, 0)\}$
2	M and 0	$\{(D_1, 0), (D_2, 0.7), (D_3, 0.3)\}$
3	L and 0	$\{(D_1, 0.45), (D_2, 0.45), (D_3, 0.1)\}$
4	S and 1	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0)\}$
5	M and 1	$\{(D_1, 0.1), (D_2, 0.45), (D_3, 0.45)\}$
6	L and 1	$\{(D_1, 0), (D_2, 0.02), (D_3, 0.98)\}$

Table 2. Online update BRB under testing influence

Rule number	Trained rule weight	$K_0(t\text{-}1)$ and $I(t)$	$K_0(t)$ distribution $\{D_1, D_2, D_3\}=\{S, M, L\}$
1	0.6995	S and 0	$\{(D_1,0.8819), (D_2,0), (D_3, 0.1811)\}$
2	1	M and 0	$\{(D_1, 0), (D_2, 0.4), (D_3, 0.6)\}$
3	0.9560	L and 0	$\{(D_1,0.\ 8662), (D_2,0), (D_3,0.\ 1338)\}$
4	0.5071	S and 1	$\{(D_1,0.475),(D_2,0.\ 213),(D_3,0.312)\}$
5	1	M and 1	{(D1, 0), (D2, 0), (D3, 1)}
6	0.9924	L and 1	$\{(D_1,0.4099),(D_2,0.1414),(D_3,0.4486)\}$

This figure gives the testing data of the zero-item drift parameter and the estimates generated by the initial BRB and the trained BRB based forecasting models.

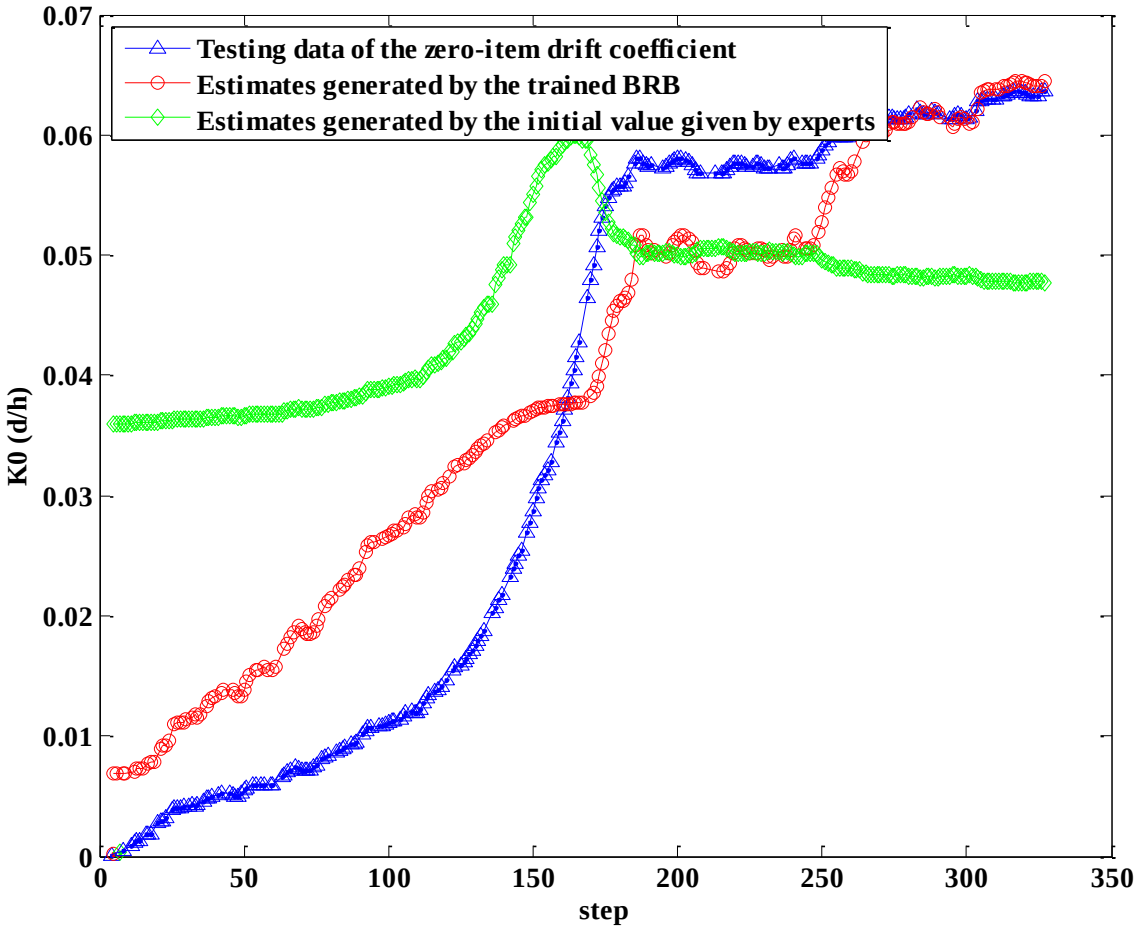


Figure. The testing data of the zero-item drift parameter and the estimates generated by the trained BRB based forecasting model

Recently published papers

1. Hu G.Y., *__Zhou Z.J.__, Zhang B.C., Yin X.J. A method for predicting the network security situation based onhidden BRB model and revised CMA-ES algorithm. Applied Soft Computing. 2016, 48: 404-418.
2. Li G.L, *__Zhou Z.J.__, Hu C.H., Chang L.L., Zhou Z.G., Zhao F.J. A new safety assessment model for complex system based on the conditional generalized minimum variance and the belief rule base. Safety Sciences. 2017, 93: 108–120
3. Zhao F.J. *__Zhou Z.J.__, Hu C.H., Chang L.L., Zhou Z.G., Li G.L. A new evidential reasoning-based method for online safety assessment of complex systems. IEEE Transactions on Systems, Man and Cybernetics: Systems. 2017, DOI: 10.1109/TSMC.2016.2630800.
4. *Chang L.L. __Zhou Z.J.__, Liao T.J., Chen Y.W., Hu Y., Yang L.B. Belief Rule Base Structure and Parameter Joint Optimization under Disjunctive assumption for Nonlinear Complex System Modeling. IEEE Transactions on Systems, Man and Cybernetics: Systems. DOI: 10.1109/TSMC.2017.2678607.
5. *__Zhou Z.J.__, Hu G.Y. Zhang B.C., Hu C.H., Zhou Z.G., Qiao P.L. A model for hidden behaviour prediction of complex systems based on belief rule base and power set. IEEE Transactions on Systems, Man and Cybernetics: Systems. (in press)
6. Chang L.L., *__Zhou Z.J.__, You Y., Yang L.B. Zhou Z.G. Belief rule based expert system for classification problems with new rule activation and weight calculation procedures. Information Sciences, 2016, 336: 75-91.
7. *__Zhou Z.J.__, Chang L.L., Hu C.H., Han X.X. Zhou Z.G. A new BRB-ER based model for assessing the life of product using data under various environments. IEEE Transactions on Systems, Man and Cybernetics: Systems. 2016, 46(11): 1529-1543.
8. *__Zhou Z.J.__, Hu C.H., Hu G.Y., Han X.X., Zhang B.C., Chen Y.W. Hidden behavior prediction of complex systems under testing influence based on semi-quantitative information and belief rule base. ***IEEE Transactions on Fuzzy Systems***, 2015, 23(6): 2371-2386.

Thank you!